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# **Chiral Magnetic Effect in the Sakai-Sugimoto model**

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work in collaboration with:

*Andreas Schmitt, Stefan Stricker, JHEP01(2010)026*

# Anomaly-induced currents in dense matter with strong $\vec{B}$ -fields

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$$\vec{J}_5 = \frac{e^2 N_c}{2\pi^2} \mu \vec{B}$$

... exact when  $\chi$  symmetry unbroken [G.M. Newman, D.T. Son, PRD73, 045006 (2006)]

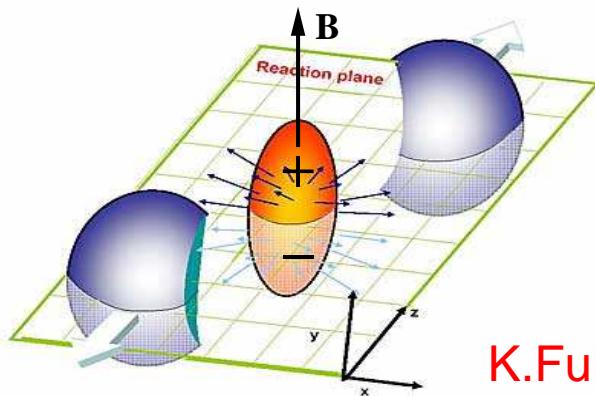
# Anomaly-induced currents in dense matter with strong $\vec{B}$ -fields

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Similarly (?) the **Chiral Magnetic Effect** (cf. yesterday's talks):



$$\vec{J}_{\text{e.m.}} = \frac{e^2 N_c}{2\pi^2} \mu_5 \vec{B}$$

$\mu_5 \leftrightarrow$  net chiral density

K.Fukushima, D.E.Kharzeev, H.J.Warringa, PRD 78, 074033 (2008)

Q: corrections from strong interactions possible?

NO, exact: A.Y. Alekseev, V.V. Cheianov, J. Fröhlich, PRL 81, 3503 (1998); FKW

YES, reduced: K. Fukushima, M. Ruggieri, arXiv:1004.2769

# Holographic QCD

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Top-down:

Holographic N=4 Super-Yang-Mills: AdS/CFT with  $N_c \rightarrow \infty$  D3 branes from type-IIB string theory ([Maldacena 1998](#))

E. Witten, *Adv. Theor. Math. Phys.* 2, 505 (1998):

Holographic *nonsupersymmetric* QCD from type-IIA string theory

with  $N_c \rightarrow \infty$  D4 branes compactified on circle  $x_4 \equiv x_4 + 2\pi/M_{KK}$

- antisymmetric b.c. for adjoint fermions: masses  $\sim M_{KK}$
- adjoint scalars not protected by gauge symmetry: also masses  $\sim M_{KK}$

→ dual to pure-glue YM theory

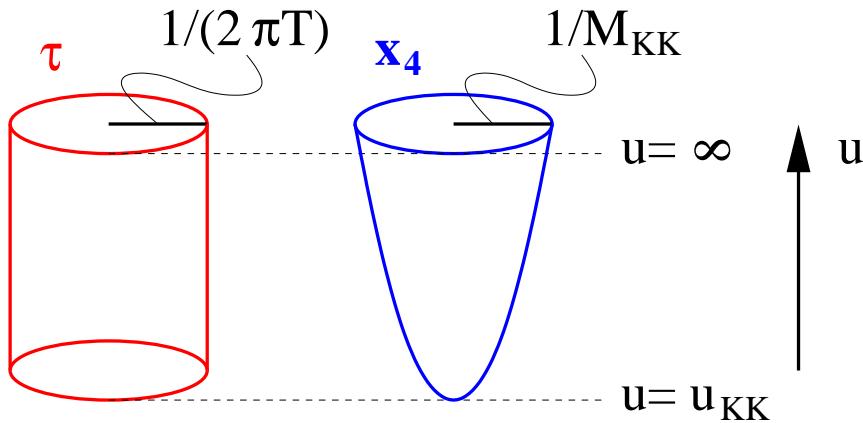
3+1-dimensional after  $M_{KK} \rightarrow \infty$ ,  $\lambda = \frac{g_5^2 N_c}{2\pi/M_{KK}} \ll 1$

but supergravity approximation needs  $\lambda \gg 1$

# Hawking-Page phase transition

## Confined phase

$$ds_{\text{conf}}^2 = \left(\frac{u}{R}\right)^{3/2} [d\tau^2 + d\mathbf{x}^2 + f(u)dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[ \frac{du^2}{f(u)} + u^2 d\Omega_4^2 \right]$$

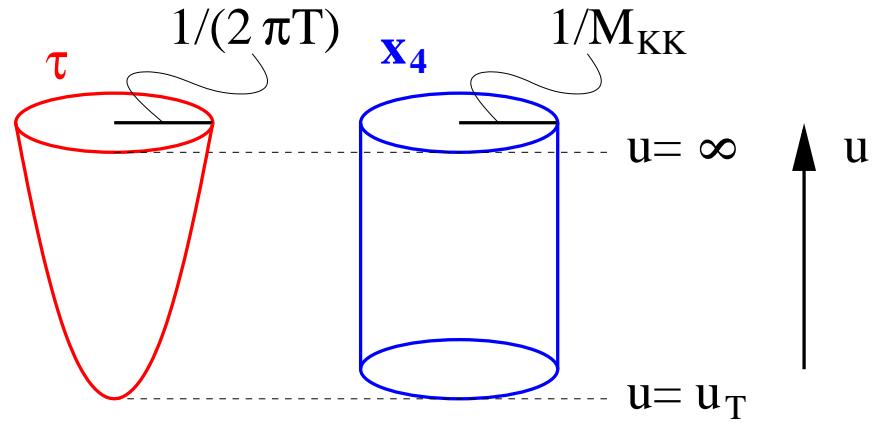


$$M_{\text{KK}} = \frac{3}{2} \frac{u_{\text{KK}}^{1/2}}{R^{3/2}} \quad f(u) \equiv 1 - \frac{u_{\text{KK}}^3}{u^3}$$

Cigar topology in  $x_4$ - $u$  subspace →

## Deconfined phase

$$ds_{\text{deconf}}^2 = \left(\frac{u}{R}\right)^{3/2} [\tilde{f}(u)d\tau^2 + \delta_{ij}d\mathbf{x}^2 + dx_4^2] + \left(\frac{R}{u}\right)^{3/2} \left[ \frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2 \right]$$



$$T = \frac{3}{4\pi} \frac{u_T^{1/2}}{R^{3/2}} \quad \tilde{f}(u) \equiv 1 - \frac{u_T^3}{u^3}$$

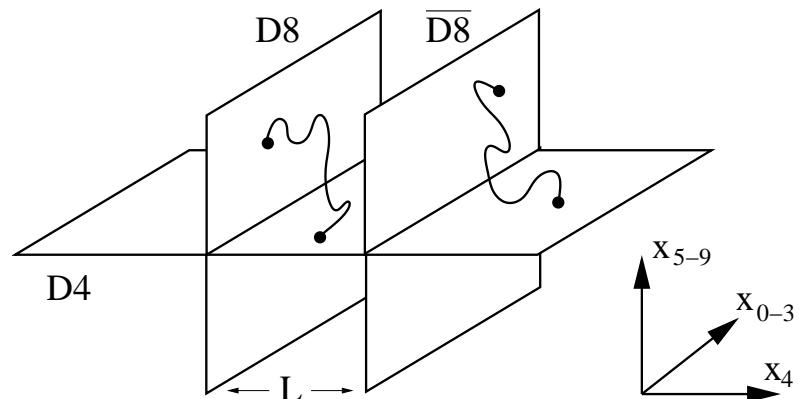
Cigar in  $\tau$ - $u$  = **Euclidean black hole**

# Sakai-Sugimoto: Adding chiral matter

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

add  $N_f$  D8- and  $\overline{\text{D}8}$ -branes, separated in  $x_4$ ,  $N_f \ll N_c$  (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/ $\overline{\text{D}8}$	x	x	x	x		x	x	x	x	x

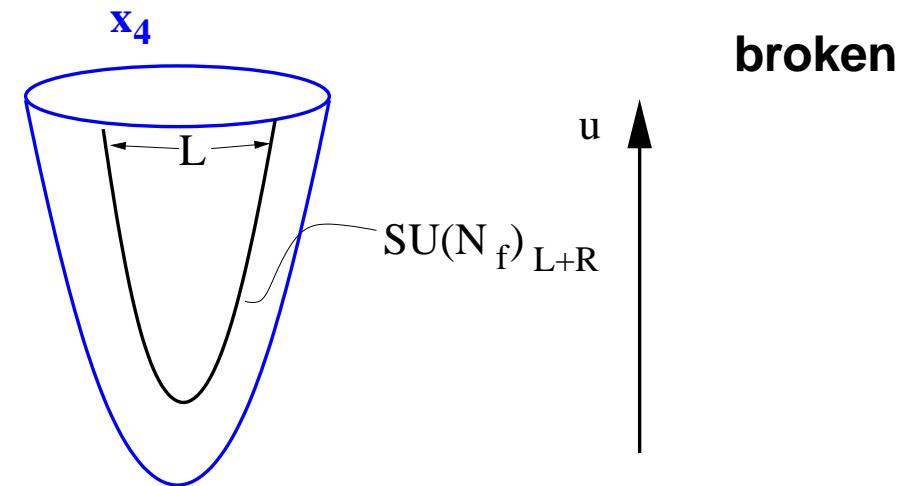
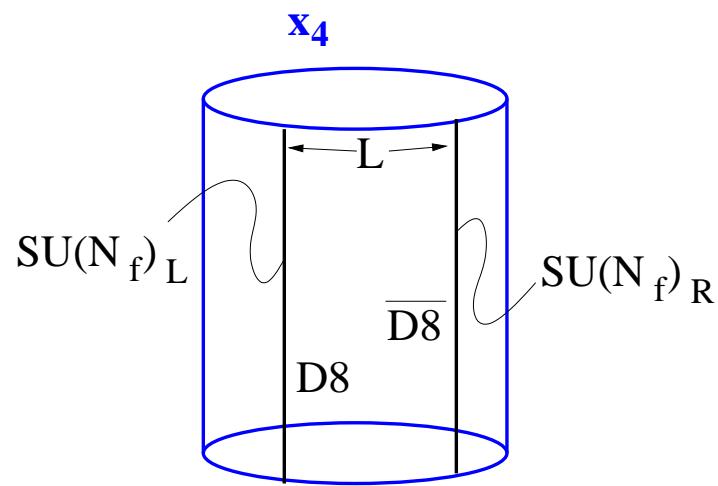


4-8, 4- $\overline{8}$  strings  
→ fundamental, massless  
chiral fermions

symmetry  $U(N_f)_L \times U(N_f)_R$   
⇒ quarks & gluons

# Chiral symmetry breaking

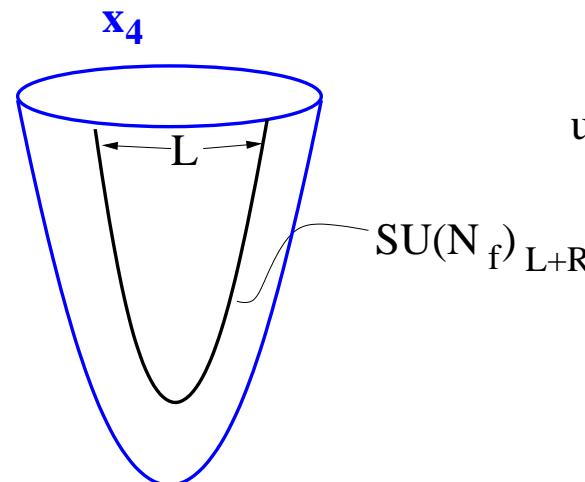
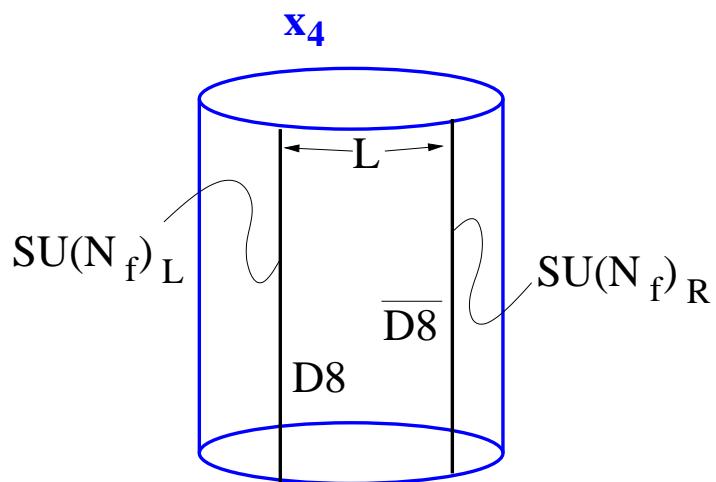
unbroken



broken

# Chiral symmetry breaking

unbroken



broken

cigar topology in  $x_4-u$  subspace:

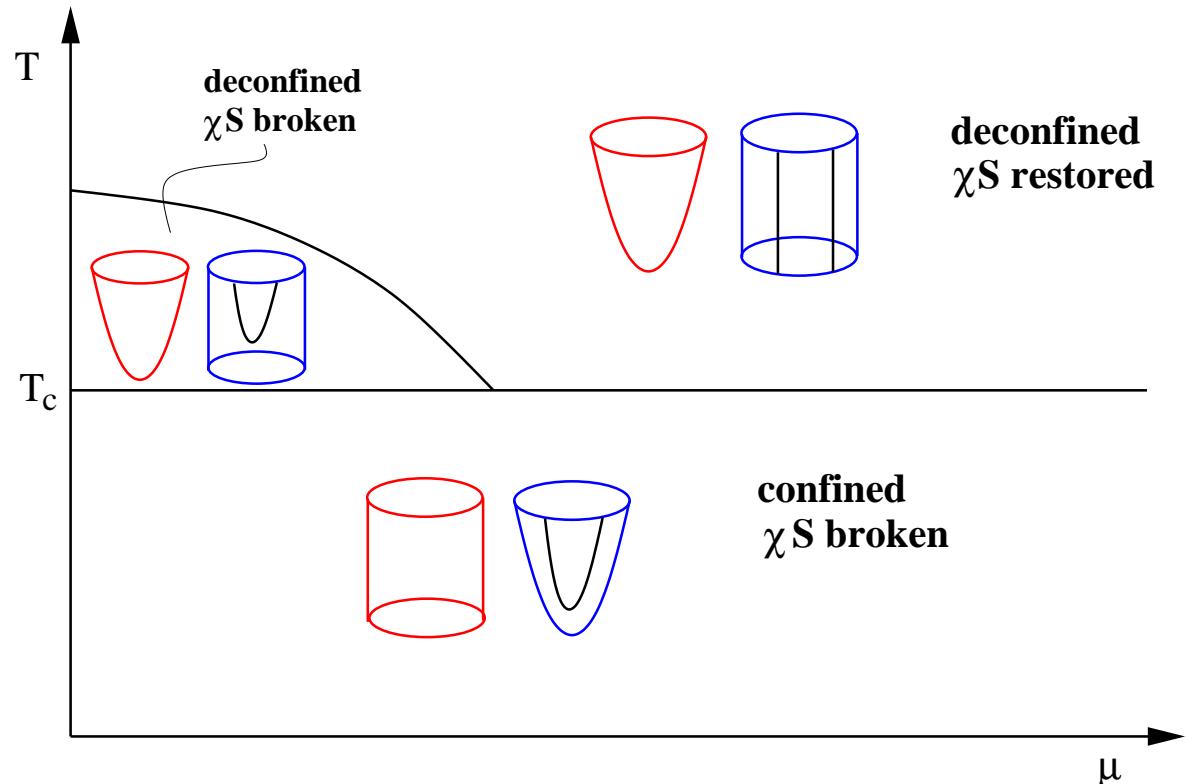
$\chi S$  necessarily broken

when  $D8-\overline{D8}$  branes not maximally separated:

possible difference between

deconfinement and  $\chi S$  restoration

in following: maximal separation only



## D8 brane action

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$$S_{\text{D8}} = S_{\text{YM}} + S_{\text{CS}}$$

$N_f = 1$  for simplicity;  $A_z = 0$  gauge

$$\begin{aligned} S_{\text{YM}} &= \kappa M_{\text{KK}}^2 \int d^4x \int_{-\infty}^{\infty} dz \left[ k(z) F_{z\mu} F^{z\mu} + \frac{h(z)}{2M_{\text{KK}}^2} F_{\mu\nu} F^{\mu\nu} \right], \\ S_{\text{CS}} &= \frac{N_c}{24\pi^2} \int d^4x \int_{-\infty}^{\infty} dz A_\mu F_{z\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}, \\ \kappa &\equiv \frac{\lambda N_c}{216\pi^3}, \quad k(z) \equiv 1 + z^2, \quad h(z) \equiv (1 + z^2)^{-1/3} \end{aligned}$$

- chemical potentials  $\mu = (\mu_R + \mu_L)/2$ ,  $\mu_5 = (\mu_R - \mu_L)/2$   
as boundary conditions  $A_0(z = \pm\infty) = \mu_{L/R}$
- external U(1) gauge field (nondynamical)  $A_1(x_2, z = \pm\infty) = -x_2 B$

following Sakai-Sugimoto:  $M_{\text{KK}} \simeq 949 \text{ MeV}$ ,  $N_c = 3$ ,  $\lambda \simeq 16.6$ ,  $\kappa \simeq 0.007$

# Chiral currents

## Field equations

$$\begin{aligned}\kappa M_{\text{KK}}^2 \partial_z [k(z) F^{z\mu}] + \kappa h(z) \partial_\nu F^{\nu\mu} &= \frac{N_c}{16\pi^2} F_{z\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}, \\ \kappa M_{\text{KK}}^2 \partial_\mu [k(z) F^{z\mu}] &= \frac{N_c}{64\pi^2} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma},\end{aligned}$$

From gospel according to Witten:

$$\mathcal{J}_{L/R}^\mu \equiv -\frac{\delta S}{\delta A_\mu(x, z = \pm\infty)} = \left( \underbrace{\mp 2\kappa M_{\text{KK}}^2 k(z) F^{z\mu}}_{\mathcal{J}_{\text{YM}}} \pm \underbrace{\frac{N_c}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}}_{\mathcal{J}_{\text{CS}}} \right)_{z=\pm\infty}$$

**NB:** when  $\mathcal{J}_{\text{CS}} \neq 0$  different than factor in normalizable mode!!

$$A_\mu(x, z) = \underbrace{A_\mu(x, z = \pm\infty)}_{\text{enters } \mathcal{J}_{\text{CS}}} \pm \frac{\mathcal{J}_{\mu, \text{YM}}^{L/R}}{2\kappa M_{\text{KK}}^2} \frac{1}{z} + \mathcal{O}\left(\frac{1}{z^2}\right)$$

# Consistent and covariant anomalies

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$$\partial_\mu \mathcal{J}_{L/R}^\mu = \partial_\mu (\mathcal{J}_{\text{YM}} + \mathcal{J}_{\text{CS}})_{L/R}^\mu = \mp \frac{N_c}{16\pi^2} \left(1 - \frac{2}{3}\right) F_{\mu\nu}^{L/R} \tilde{F}_{L/R}^{\mu\nu},$$

*consistent* anomaly (left and right separate; symmetric in V and A) [Bardeen 1969]

$$\partial_\mu \mathcal{J}_5^\mu = \frac{N_c}{\boxed{24}\pi^2} \left( F_{\mu\nu}^V \tilde{F}_V^{\mu\nu} + F_{\mu\nu}^A \tilde{F}_A^{\mu\nu} \right)$$

$$\partial_\mu \mathcal{J}^\mu = \frac{N_c}{12\pi^2} F_{\mu\nu}^V \tilde{F}_A^{\mu\nu} \not\equiv 0,$$

## Bardeen's counterterm

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$$\Delta S = c \int d^4x (A_\mu^L A_\nu^R F_{\rho\sigma}^L + A_\mu^L A_\nu^R F_{\rho\sigma}^R) \epsilon^{\mu\nu\rho\sigma},$$

$\Delta S$  is metric independent, can be written as holographic (surface) counter term  
→ renormalized left- and right-handed currents

$$\bar{\mathcal{J}}_{L/R}^\mu \equiv \mathcal{J}_{L/R}^\mu + \Delta \mathcal{J}_{L/R}^\mu, \text{ similarly } \bar{\mathcal{J}}_\mu, \bar{\mathcal{J}}_5^\mu$$

Choosing  $c = \frac{N_c}{48\pi^2}$  leads to the *covariant* anomaly

$$\begin{aligned}\partial_\mu \bar{\mathcal{J}}^\mu &= \boxed{0}, \\ \partial_\mu \bar{\mathcal{J}}_5^\mu &= \frac{N_c}{\boxed{8}\pi^2} F_{\mu\nu}^V \tilde{F}_V^{\mu\nu} + \frac{N_c}{24\pi^2} F_{\mu\nu}^A \tilde{F}_A^{\mu\nu}.\end{aligned}$$

[Adler, Bell, Jackiw, 1969]

## Only YM current

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If only YM part of current taken (occasionally done without justification), find:

$$\partial_\mu \mathcal{J}_{\text{YM},L/R}^\mu = 3 \times \partial_\mu (\mathcal{J}_{\text{YM}} + \mathcal{J}_{\text{CS}})_{L/R}^\mu$$

thus same equations as with consistent anomaly but times 3:

$$\begin{aligned}\partial_\mu \mathcal{J}_{\text{YM}}^\mu &= \frac{N_c}{4\pi^2} F_{\mu\nu}^V \tilde{F}_A^{\mu\nu}, \\ \partial_\mu \mathcal{J}_{\text{YM},5}^\mu &= \frac{N_c}{\boxed{8}\pi^2} \left( F_{\mu\nu}^V \tilde{F}_V^{\mu\nu} + F_{\mu\nu}^A \tilde{F}_A^{\mu\nu} \right),\end{aligned}$$

Coincides with covariant anomaly for  $F_A = 0$ , but  $\partial_\mu \mathcal{J}^\mu \not\equiv 0$  !

OK as long as  $F_A = 0$  ?

But need  $\vec{\nabla} \mu_5 \sim \vec{E}_A$  for charge separation at RHIC!?

$$\partial_t \rho = -\nabla \mathcal{J} \propto B \nabla \mu_5$$

# Ansatz and results for bulk gauge fields

boundary conditions  $A_0(z = \pm\infty) = \pm\mu_5$  (when  $\mu = 0$ ),

$$A_1(z = \pm\infty, x_2) = -x_2 B$$

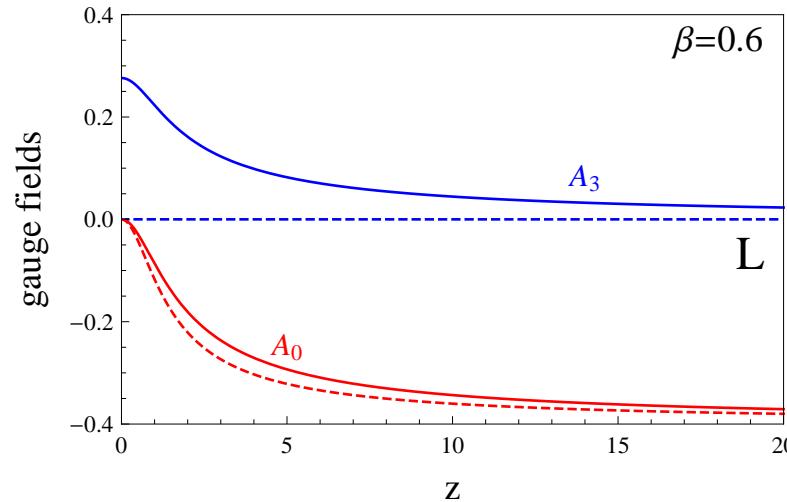
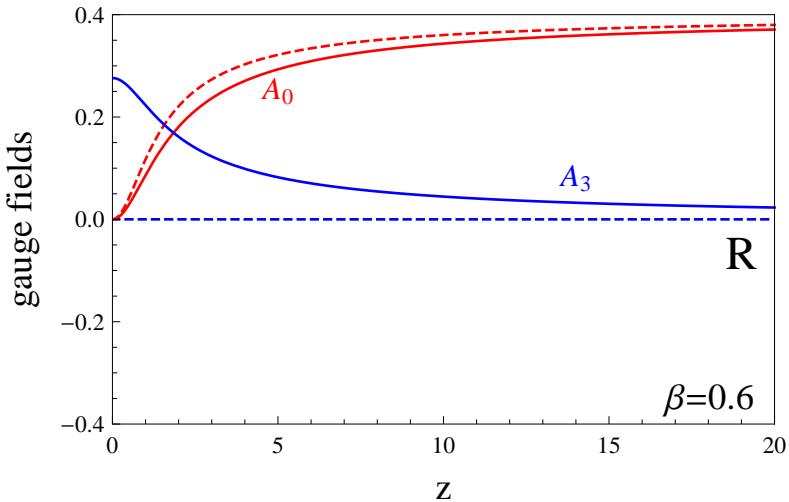
Ansatz: nonzero (brane) gauge fields with dependence on  $z$ :  $A_0(z)$ ,  $A_3(z)$

→ magnetic field constant in bulk:  $A_1(x_2) = -x_2 B$

→ closed-form solutions with YM approximation to DBI action

(good for small and also very large  $B$ )

e.g. chirally restored phase,  $\mu = 0$ ,  $\mu_5 \neq 0$ ,  $\beta \equiv \frac{27\pi B}{2\lambda M_{KK}^2} \simeq \frac{B}{2 \cdot 10^{19} G}$



(dashed is  $\beta = 0$ ; full is  $\beta = 0.6$ , where  $A_3 \not\equiv 0 \leftrightarrow \mathcal{J}_3$ )

# Results for currents

[A. Rebhan, A. Schmitt, S.A. Stricker, JHEP 1001, 026 (2010)]

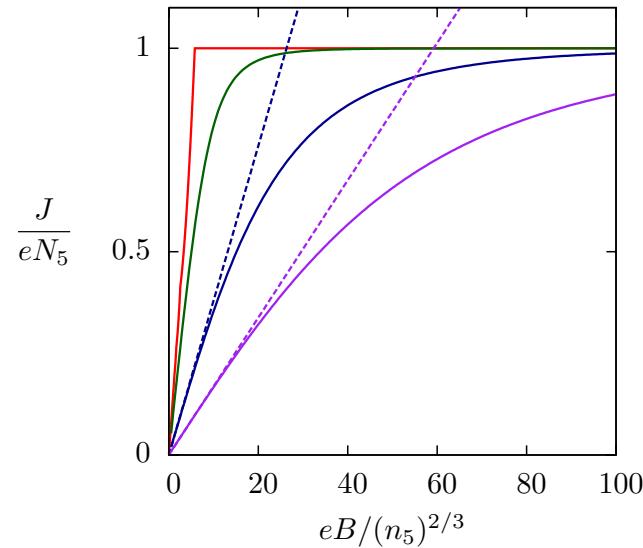
	$\mathcal{J}_{\text{YM}}$	$\mathcal{J}_{\text{YM+CS}}$	$\mathcal{J}_{\text{YM+CS}} + \Delta\mathcal{J}$	$\mathcal{J}'_{\text{YM+CS}}$
anomaly	“semi-covariant”:	consistent:	<u>covariant:</u>	absent:
$\partial_\mu \mathcal{J}_5^\mu / \frac{N_c}{24\pi^2}$	$3F_V \tilde{F}_V + 3F_A \tilde{F}_A$	$F_V \tilde{F}_V + F_A \tilde{F}_A$	$3F_V \tilde{F}_V + F_A \tilde{F}_A$	0
$\partial_\mu \mathcal{J}^\mu / \frac{N_c}{24\pi^2}$	$6F_V \tilde{F}_A$	$2F_V \tilde{F}_A$	$\underline{0}$	0
$(\mathcal{J}_\parallel^5 / \frac{\mu B N_c}{2\pi^2}) \Big _{T>T_c}$	1	$\frac{2}{3}$	$1 = 1 - \frac{1}{3} + \frac{1}{3}$	$\frac{1}{2}$
<b>CME:</b> $\mathcal{J}_\parallel / \frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	$0 = 1 - \frac{1}{3} - \frac{2}{3}$	$\frac{1}{2}$

$\mathcal{J}'_{\text{YM+CS}}$  corresponds to ad-hoc modified CS action enforcing thermodynamic consistency **Bergman, Lifshytz, Lippert 08**  
 however, eliminates anomaly! **RSS 2010**

# Results for currents

	$\mathcal{J}_{\text{YM}}$	$\mathcal{J}_{\text{YM+CS}}$	$\mathcal{J}_{\text{YM+CS}} + \Delta\mathcal{J}$
anomaly	“semi-covariant”	consistent	<u>covariant</u>
$(\mathcal{J}_{\parallel}/\mathcal{J}_5^0) _{B \rightarrow \infty}$	1	$\frac{2}{3}$	0

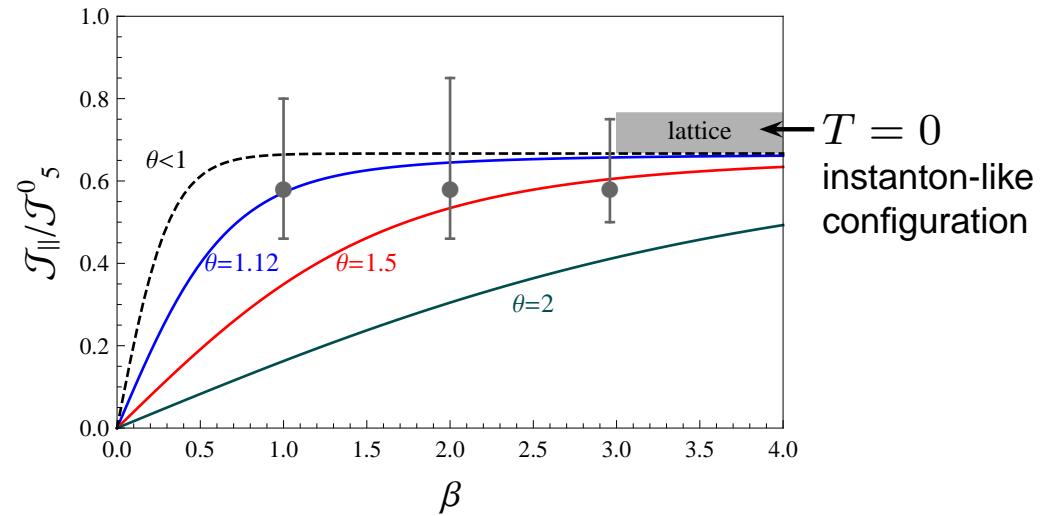
like in weak coupling:



K.Fukushima *et al.*, PRD 78, 074033 (2008)

like on the lattice?:

$$\theta = T/T_c = 1.12$$



P. V. Buividovich *et al.*, PRD 80, 054503 (2009)

AR, A. Schmitt, S.A. Stricker, JHEP 1001, 026 (2010)

# Questions

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- Time dependence of  $B$ !  
Need frequency-dependent chiral magnetic conductivity?  
Only static DC component should play a role in charge separation!
- Is  $\mathcal{J}_{\text{YM}}$  (normalizable mode in gauge field) the more appropriate quantity?  
But with  $\nabla \mu_5 \neq 0$  as needed for actual charge separation  
( $\partial_t \rho = -\nabla \mathcal{J} \propto B \nabla \mu_5$  when  $B$  homogeneous)  
 $\partial_\mu \mathcal{J}_{\text{YM}}^\mu \neq 0$
- Does  $\mathcal{J}$  in contrast to  $\mathcal{J}_5$  receive (strong) corrections?  
Fukushima & Ruggieri:  $\mathcal{J} = (\text{top.result})/(1 + g^2 \dots) \rightarrow 0$  for  $g \rightarrow \infty$  ?
- Could SS result for renormalized current  $\bar{\mathcal{J}}$  be even right?  
E.g., through absence of quasiparticles at infinite coupling?
- ...